5 Logic and Inference: Rules

He is unable to pay more than \$ 400 in total. If given the choice, he would go for the cheapest option. His second priority is the presence of a garden; his lowest priority is additional space.

5.6.1 Formalization of Carlos's Requirements

We use the following predicates to describe properties of apartments:

size(x,y)	y is the size of apartment x (in sq m)					
bedrooms(x, y)	x has y bedrooms					
price(x,y)	<i>y</i> is the price for <i>x</i>					
floor(x, y)	<i>x</i> is on the <i>y</i> th floor					
gardenSize(x,y)	x has a garden of size y					
lift(x)	there is an elevator in the house of x					
pets(x)	pets are allowed in <i>x</i>					
central(x)	x is centrally located					

We also make use of the following predicates:

acceptable(x)	flat x satisfies Carlos's requirements				
offer(x,y)	Carlos is willing to pay y for flat x				

Now we present Carlos's firm requirements. Any apartment is a priori acceptable.

 $r_1 : \Rightarrow acceptable(X)$

However, *Y* is unacceptable if one of Carlos's requirements is not met.

$$\begin{split} r_{2} &: bedrooms(X,Y), Y < 2 \Rightarrow \neg acceptable(X) \\ r_{3} &: size(X,Y), Y < 45 \Rightarrow \neg acceptable(X) \\ r_{4} &: \neg pets(X) \Rightarrow \neg acceptable(X) \\ r_{5} &: floor(X,Y), Y > 2, \neg lift(X) \Rightarrow \neg acceptable(X) \\ r_{6} &: price(X,Y), Y > 400 \Rightarrow \neg acceptable(X) \end{split}$$

Rules r_2 - r_6 are exceptions to rule r_1 , so we add

 $r_2 > r_1, r_3 > r_1, r_4 > r_1, r_5 > r_1, r_6 > r_1$

5.6 Example of Nonmonotonic Rules: Brokered Trade

Next we calculate the price Carlos is willing to pay for an apartment.

$$r_{7}: size(X,Y), Y \ge 45, garden(X,Z), central(X) \Rightarrow offer(X, 300 + 2Z + 5(Y - 45))$$

$$r_{8}: size(X,Y), Y \ge 45, garden(X,Z), \neg central(X) \Rightarrow offer(X, 250 + 2Z + 5(Y - 45))$$

An apartment is only acceptable if the amount Carlos is willing to pay is not less than the price specified by the landlord (we assume no bargaining can take place).

$$r_9: \ offer(X,Y), price(X,Z), Y < Z \Rightarrow \neg acceptable(X) \\ r_9 > r_1$$

5.6.2 Representation of Available Apartments

Each available apartment is given a unique name, and its properties are represented as facts. For example, apartment a_1 might be described as follows:

 $bedrooms(a_1,1)$ $size(a_1,50)$ $central(a_1)$ $floor(a_1,1)$ $\neg lift(a_1)$ $pets(a_1)$ $garden(a_1,0)$ $price(a_1,300)$

The description of the available apartments are summarized in table 5.1. In practice, the flats on offer could be stored in a relational database.

If we match Carlos's requirements and the available apartments, we see that

- flat a_1 is not acceptable because it has one bedroom only (rule r_2)
- flats a_4 and a_6 are unacceptable because pets are not allowed (rule r_4)
- for a_2 , Carlos is willing to pay \$ 300, but the price is higher (rules r_7 and r_9)
- flats a_3, a_5 , and a_7 are acceptable (rule r_1)

5 Logic and Inference: Rules

Flat	Bedrooms	Size	Central	Floor	Lift	Pets	Garden	Price
a_1	1	50	yes	1	no	yes	0	300
a_2	2	45	yes	0	no	yes	0	335
a_3	2	65	no	2	no	yes	0	350
a_4	2	55	no	1	yes	no	15	330
a_5	3	55	yes	0	no	yes	15	350
a_6	2	60	yes	3	no	no	0	370
a_7	3	65	yes	1	no	yes	12	375



5.6.3 Selecting an Apartment

So far we have identified the apartments acceptable to Carlos. This selection is valuable in itself, since it reduces the focus to relevant flats, which may then be physically inspected. But it is also possible to reduce the number further, even down to a single apartment, by taking further preferences into account. Carlos's preferences are based on price, garden size, and size, in that order. We represent them as follows:

$$\begin{split} r_{10}: cheapest(X) &\Rightarrow rent(X) \\ r_{11}: cheapest(X), largestGarden(X) &\Rightarrow rent(X) \\ r_{12}: cheapest(X), largestGarden(X), largest(X) &\Rightarrow rent(X) \\ r_{12} &> r_{10} \\ r_{12} &> r_{11} \\ r_{11} &> r_{10} \end{split}$$

Also, we need to specify that at most one apartment can be rented, using conflict sets:

$$C(rent(x)) = \{\neg rent(x)\} \cup \{rent(y) \mid y \neq x\}$$

The prerequisites of these rules can be derived from the set of acceptable apartments using further rules. Here we keep the discussion simple by just stating the facts for our example:

 $cheapest(a_3)$

5.7 Rule Markup in XML: Monotonic Rules

```
cheapest(a_5)
largest(a_3)
largest(a_7)
largestGarden(a_5)
```

Now the theory is able to derive the decision to rent a_5 :

- Rule r_{11} can be applied to a_5 .
- Rule r_{10} can be applied to a_3 , thus establishing an attack. However, this attack is successfully countered because r_{11} is stronger than r_{10} .
- This is indeed the only attack, because neither r_{11} nor r_{12} applies to any other apartment.

Thus a selection has been made, and Carlos will soon move in.

5.7 Rule Markup in XML: Monotonic Rules

Our aim here is to make knowledge in the form of rules machine-accessible, in accordance with the Semantic Web vision. We outline an encoding of monotonic rules in XML.

5.7.1 Terms

Terms are represented using XML tags <term>, <function>, <var>, and <const>. For example, the term

f(X, a, g(b, Y))

is represented as follows: