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Morphological Granulometries for Color Images

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Abstract: In this paper a new morphological technique for size and shape analysis of color granular images is presented. The method is based on the sieving model reported by Matheron. According to this model, granular images can be characterised by means of the manner in which they are sieved through various size and shape sieves. The morphological operations that are used in this analysis are defined by means of a new ordering of vectors (colors) of the HSV color space. Minimum and maximum operators are defined, and the fundamental morphological operations are extracted. The proposed method was tested with a variety of images and such experimental results are provided.

1 Introduction

Granulometries are parameterised families of morphological openings that are used for pattern and texture classification and for granular filtering. The most basic class of binary granulometries is composed of unions of openings by structuring elements that are scaled by a single parameter. Matheron [1] developed a morphological method for characterising granular images based on sieving. In this sieving model, granular images are sieved through various size and shape sieves [2]. If an image is considered as a collection of grains, then whether or not an individual grain will pass through the sieve depends on its size and shape relative to the mesh of the sieve. If we keep the basic shape and we increase the size of the mesh, more and more of the image will pass through, the eventual result being that no more grains remain. This model serves as a means to approach the removal of nonconforming image structure. Additionally, it can be further developed to obtain image signatures based on the rate of sieving.

In this paper a new granulometric technique suitable for size and shape analysis of color granular images is introduced. The proposed method uses vector morphological operations to extend the classical sieving model in order to accommodate color images. Initially, the basic concepts of granulometries are briefly reviewed. Subsequently, the vector ordering that is used for the definitions of the morphological operations is introduced. By means of this ordering, minimum and maximum operators are presented and the definitions of the vector morphological operations that are used for the granulometric analysis of color images are derived. The proposed morphological operators are vector preserving and possess the same basic properties with their gray-scale counterparts. Illustrative examples of the application of the method in color granular images are provided.

2 Granulometries

The basic type of granulometries are the Euclidean granulometries which can be defined as follows [1]. If we consider $t > 0$ as a variable, the class of images {A o tB}, where o denotes the opening operation, is called a granulometry and the primitive B is said to be a generator of the granulometry. If $\Omega(t)$ is the area of A o tB, with $\Omega(0)$ being the area of A itself, then $\Omega(t)$ is a decreasing function of t and is called a size distribution. A normalised size distribution is defined by:

$$\Phi(t) = 1 - \Omega(t) / \Omega(0) . \quad (1)$$

$\Phi(t)$ increases from 0 to 1 and can be shown to be a probability distribution function. Thus, its derivative $d\Phi(t)$ is a probability density. In the terminology of Matheron, both Φ and $d\Phi$ are known as granulometric size distributions. More recently, they have come to be known as the pattern spectrum of the image relative to the granulometry (or, relative to the generator). The moments of $d\Phi$ can be employed as image signatures.

This method can be generalized in order to be applicable to digital images in the following way: we consider a sequence $\{E_k\}$, $k = 1, 2, \ldots$, of structuring elements of increasing size, where E_{k+1} is E_k – open for all k. Owing to the latter requirement, if S is any image, then S o E_{k+1} is a subimage of S o E_k . Consequently, opening in turn by the structuring elements yields a decreasing sequence of images. Let $\Omega(k)$ be the number of pixels in S o E_k . Then, $\Omega(k)$ is a decreasing function of k and, assuming that E_1 consists of a single pixel, $\Omega(1)$ gives the original pixel count in S. In a similar way, the normalized size distribution $\Phi(k)$ is defined by:

$$\Phi(k) = 1 - \Omega(k) / \Omega(1) . \quad (2)$$

This is a probability distribution function and its derivative, $d\Phi(k) = \Phi(k + 1) - \Phi(k)$, is a probability density. Again, the density is called a granulometric size distribution or pattern spectrum, and its moments can be employed as image signatures.

3 Vector Ordering in the HSV Color Space

The extension of the concepts of gray-scale morphology to color images implies the definition of an appropriate ordering of vectors (colors) in a concrete color space [3, 4, 5]. The HSV color space has been chosen since it is closely related to the way in which humans perceive color. In this color space each color is a vector with three components: h (Hue), s (Saturation), v (Value), with $h \in [0,360)$, $s \in [0,1]$, $v \in [0,1]$.

The proposed vector ordering scheme is as follows [6]:

(i) Initially, vectors are ordered with respect to the third component v. More specifically, they are sorted from vectors with the smallest v to vectors with the greatest v.

(ii) Vectors having the same value of v are ordered with respect to the second component s. Particularly, they are sorted from vectors with the greatest s to vectors with the smallest s.

(iii) Finally, vectors that have the same values of s and v are ordered with respect to the h component. More specifically, they are sorted from vectors with the smallest h to vectors with the greatest h.

Let S_n be a subset of the HSV space, which includes n vectors $x_1(h_1,s_1,v_1)$, $x_2(h_2,s_2,v_2),\ldots, x_n(h_n,s_n,v_n)$. Using the previously vector ordering procedure, we define the $\wedge$ minimum operator in S_n as follows:

$$\wedge S_n = \wedge \{ x_1(h_1,s_1,v_1),\ x_2(h_2,s_2,v_2),\ldots, x_n(h_n,s_n,v_n) \} =$$

$$= x_k(h_k,s_k,v_k): \begin{cases} v_k = \min\{v_1,v_2,\ldots,v_n\} \text{ if } \nexists\, i \neq j : v_i=v_j=\min\{v_1,v_2,\ldots,v_n\} \\ \quad\text{or} \\ v_k = v_i = v_j = \min\{v_1,v_2,\ldots,v_n\} \text{ and } s_k = \max\{s_i,s_j\} \\ \text{if } \exists\, i \neq j : v_i=v_j=\min\{v_1,v_2,\ldots,v_n\} \\ \quad\text{or} \\ v_k = v_i = v_j = \min\{v_1,v_2,\ldots,v_n\} \text{ and } s_k=s_i=s_j \\ \text{and } h_k = \min\{h_i,h_j\} \text{ if } \exists\, i \neq j : v_i=v_j=\min\{v_1,v_2,\ldots,v_n\} \\ \text{and } s_i=s_j \end{cases} \quad (3)$$

with $1\leq k \leq n$, $1\leq i, j \leq n$.

In a similar way we define the $\vee$ maximum operator in S_n as follows:

$$\vee S_n = \vee \{ x_1(h_1,s_1,v_1),\ x_2(h_2,s_2,v_2),\ldots, x_n(h_n,s_n,v_n) \} =$$

$$= x_k(h_k,s_k,v_k): \begin{cases} v_k = \max\{v_1,v_2,\ldots,v_n\} \text{ if } \nexists\, i \neq j : v_i=v_j=\max\{v_1,v_2,\ldots,v_n\} \\ \quad\text{or} \\ v_k = v_i = v_j = \max\{v_1,v_2,\ldots,v_n\} \text{ and } s_k = \min\{s_i,s_j\} \\ \text{if } \exists\, i \neq j : v_i=v_j=\max\{v_1,v_2,\ldots,v_n\} \\ \quad\text{or} \\ v_k = v_i = v_j = \max\{v_1,v_2,\ldots,v_n\} \text{ and } s_k=s_i=s_j \\ \text{and } h_k = \max\{h_i,h_j\} \text{ if } \exists\, i \neq j : v_i=v_j=\max\{v_1,v_2,\ldots,v_n\} \\ \text{and } s_i=s_j \end{cases} \quad (4)$$

with $1\leq k \leq n$, $1\leq i, j \leq n$.

It is important to notice that the proposed operators are *vector preserving* since no vector (color), which is not present in the input data, is generated [5].

4 Morphological Operators for Color Images

4.1 Vector Erosion

Let us consider the set f to be a color image with pixel values in the HSV color space and the set g to be the structuring element for the vector morphological operations that will be described here [6].
We define vector erosion of f by g at a point x as follows:

$$(f\Theta g)(x)=\wedge\{f(z)-g_x(z)\}, \text{ for } z \in D[f] \cap D[g_x] . \quad (5)$$

As a result of the previous definition, in order to perform the vector erosion of an input image f by the structuring element g at a point x:

(i) First, we translate spatially g by x, so that its origin is located at point x.
(ii) We find all *differences* between colors of the points of f with the colors of the corresponding points of the translated g, $\forall z \in D[f] \cap D[g_x]$, where $D[f]$ is the finite domain of f.
(iii) The result of the previous step is a set of colors. We find the minimum of these colors using the new minimum operator that was defined previously. This minimum color is the color of the eroded image at the point x.

4.2 Vector Dilation

We define vector dilation of f by g at a point x as follows:

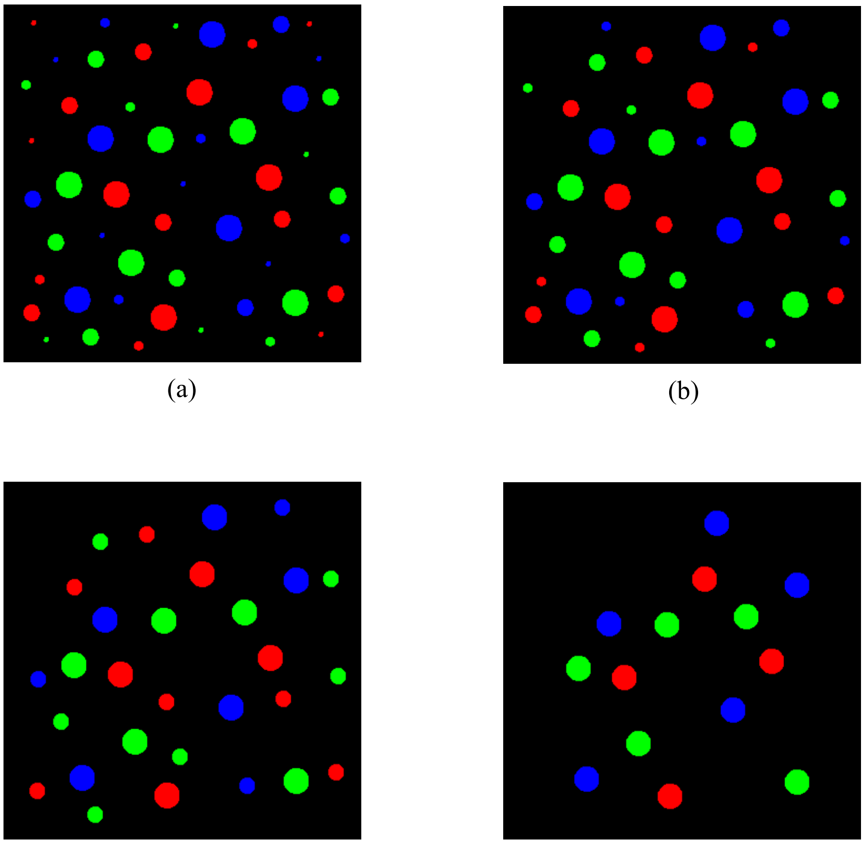
$$(f\oplus g)(x) = \vee\{f(z) + g_{-x}(-z)\}, \text{ for } z \in D[f] \cap D[g'_{-x}] . \quad (6)$$

Vector dilation is performed in a similar way to vector erosion, according to equation (6). We should notice that the operations of opening and closing are defined in the same way as their gray scale counterparts. Thus, the opening operation is defined as an erosion followed by a dilation and the closing operation as a dilation followed by an erosion.

5 Experimental Results

We have carried out a number of experiments in order to assess the performance of the new morphological technique for size and shape analysis of color granular images, using the proposed vector morphological operators. The results of these experiments for two typical color images are demonstrated below.

Let us consider Fig. 1(a), in which circles of four different sizes and of three different colors are randomly dispersed about the image. The generating sequence $\{E_k\}$ consists of circles of increasing size (the first being a single pixel). In Fig. 1(b), 1(c) and 1(d) the sieving process is clearly depicted. As larger circles are employed for the opening structuring elements, the circles in the image are successively sieved from the image. As the structuring element sequence passes each of the four sizes of the circles, the corresponding circles are sieved from the image, the result being the unnormalized size distribution $\Omega(k)$ in Fig. 2(a). The normalized size distribution $\Phi(k)$ and its derivative, the granulometric size distribution (or pattern spectrum) $d\Phi(k)$, are also depicted in Fig. 2(b) and 2(c), respectively. We should notice that the granulometric size distribution consists of four impulses that correspond to the four circle sizes and their heights correspond to the relative image areas sieved at the four stages of the granulometry in which they were eliminated. Furthermore, the representation of the image in the HSV color space allows us to discriminate circles with different hues. In Fig. 3(a), 3(b), 3(c) and 3(d) the sieving process for the red circles is demonstrated.



(a) (b) (c) (d)

Fig. 1. (a) Color image "circles", (b) Opening for k=5, (c) Opening for k=9 and (d) Opening for k=16

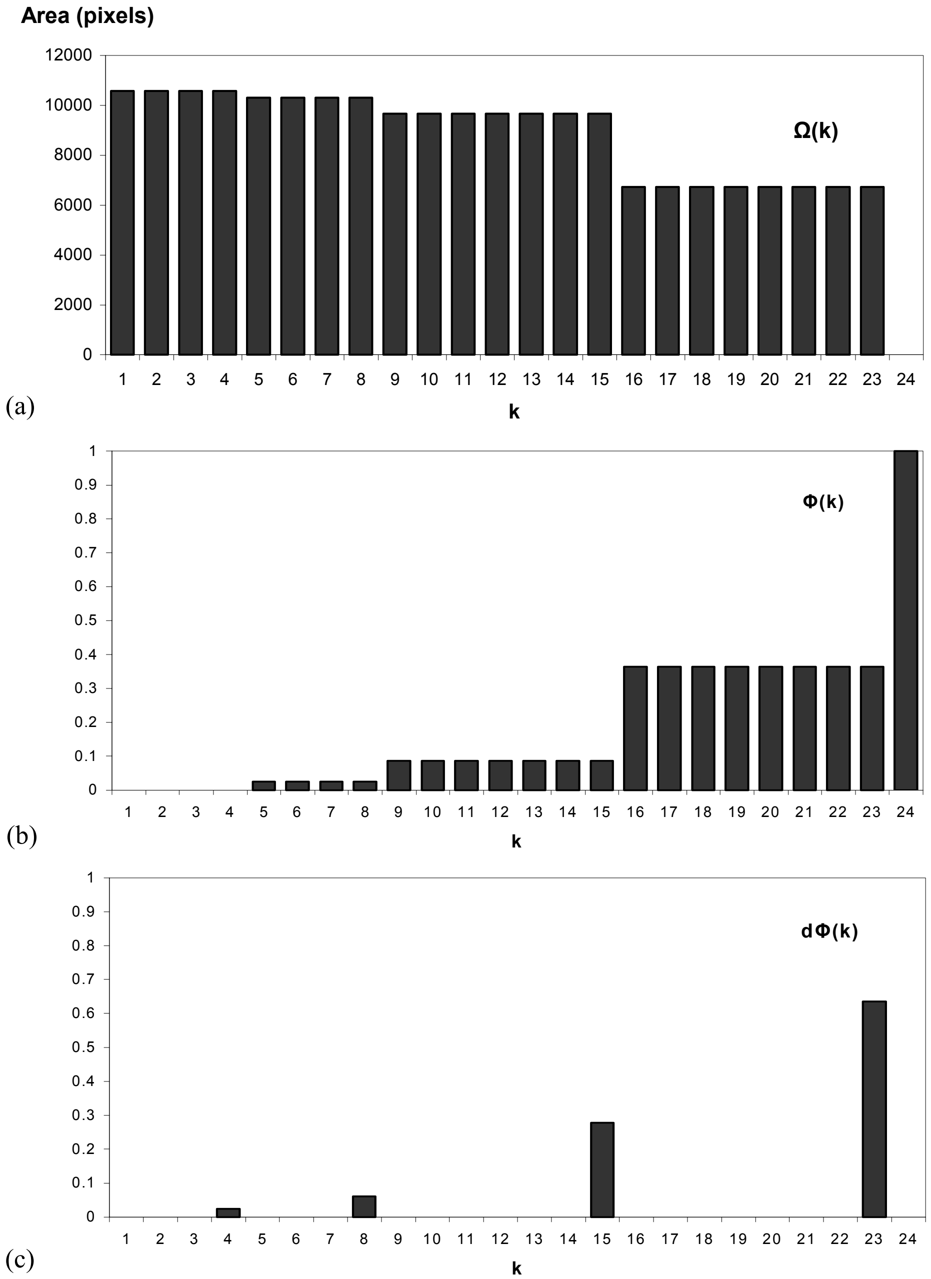


Fig. 2. (a) Unnormalized size distribution $\Omega(k)$ for color image "circles", (b) Normalized size distribution $\Phi(k)$ and (c) Granulometric size distribution $d\Phi(k)$

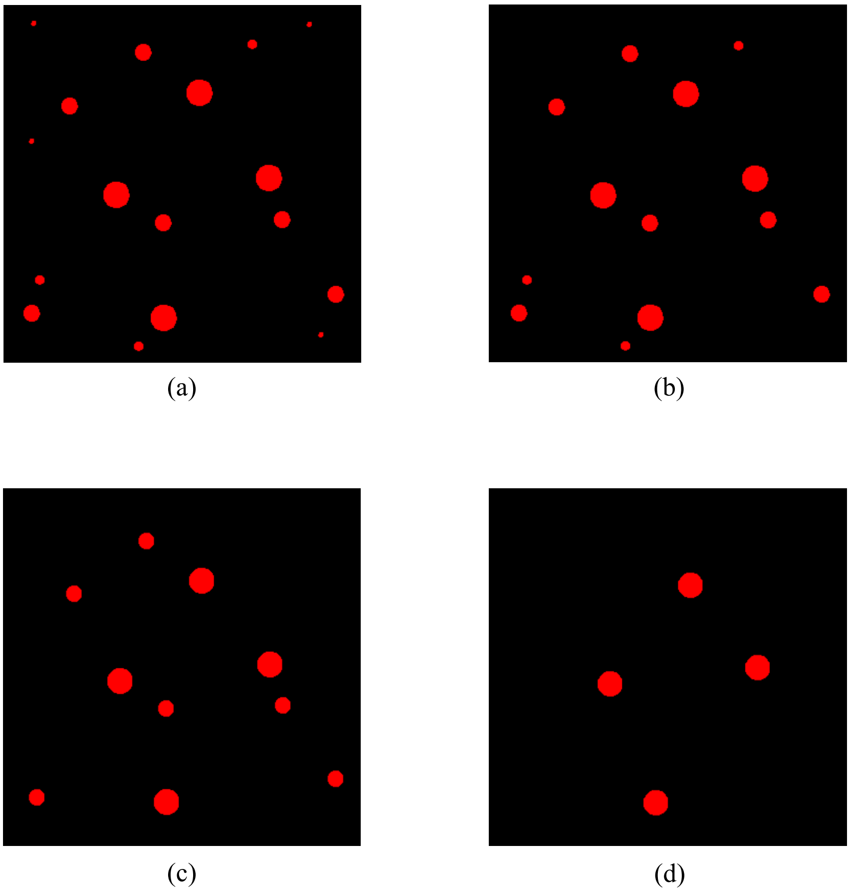


Fig. 3. (a) Red "circles", (b) Opening for k=5, (c) Opening for k=9 and (d) Opening for k=16

In Fig. 4(a) rectangles of four different sizes and of four different colors are randomly dispersed about the image, on a colored background. This is another interesting case of a granular image. The advantage of the use of morphological operators is clear. The rectangle shaped sieve can be constructed simply with the use of an appropriate structuring element. In a similar way, the sieving process for this case is demonstrated in Fig. 4(b), 4(c) and 4(d). As larger rectangles are employed for the opening structuring elements, the rectangles in the image are successively sieved from the image.

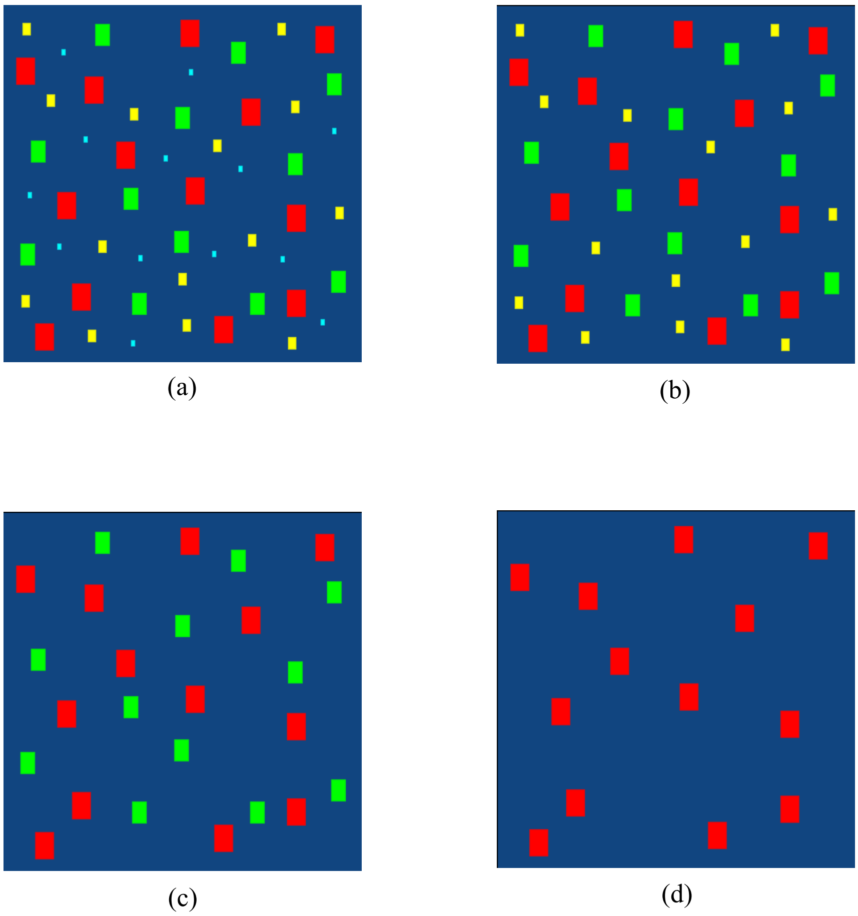
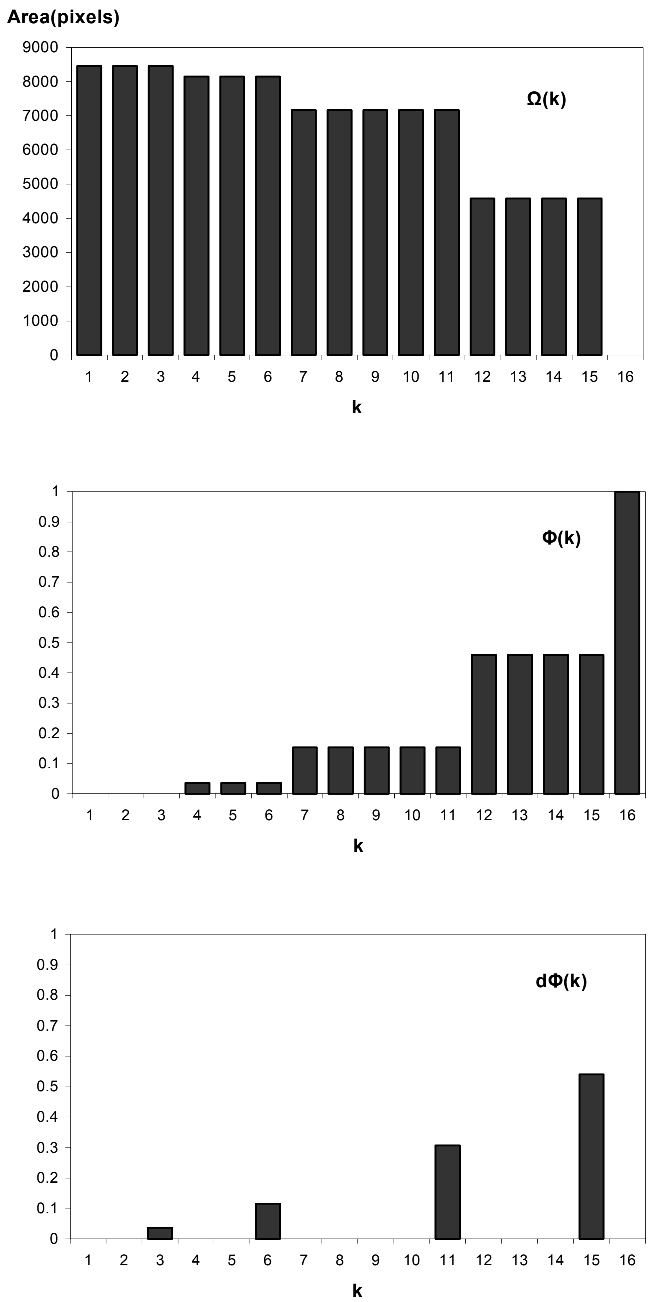


Fig. 4. (a) Color image "rectangles", (b) Opening for k=4, (c) Opening for k=7 and (d) Opening for k=12

The unnormalized size distribution $\Omega(k)$ for this example is depicted in Fig. 5(a). The normalized size distribution $\Phi(k)$ and the granulometric size distribution $d\Phi(k)$, are also depicted in Fig. 5(b) and 5(c), respectively. Again, we notice that the granulometric size distribution consists of four impulses that correspond to the four rectangle sizes, and their heights correspond to the relative image areas sieved at the four stages of the granulometry in which they were eliminated.



(a)

(b)

(c)

Fig. 5. (a) Unnormalized size distribution $\Omega(k)$ for color image "rectangles", (b) Normalized size distribution $\Phi(k)$ and (c) Granulometric size distribution $d\Phi(k)$

6 Conclusions

We have presented a new technique for the analysis of color granular images, based on the morphological characterization of sieving. We have defined the basic morphological operations of erosion, dilation, opening and closing. Finally, we have provided experimental results that illustrate the granular analysis for color images with the corresponding size distributions and pattern spectrums. Besides application to grains and particles, the method is also effective for texture and shape analysis.

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